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TECHNICAL NOTES

Condensation on a vertical plate fin of variable thickness

P. K. SARMA and S. P. CHARY
Department of Mechanical Engineering, College of Engineering, Andhra University, Visakhapatnam 530003, India

and

V. DHARMA RAO
Department of Chemical Engineering, College of Engineering, Andhra University, Visakhapatnam 530003, India

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STARTING with the work of Nusselt, the process of film condensation has been extensively investigated both experimentally and analytically. Merte [1] presented the state-of-the-art in his review article. More recently, condensation on different geometric configurations and extended surfaces has been studied. Patankar and Sparrow [2] solved the problem of condensation on an extended surface by considering the heat conduction in the fin as a two-dimensional phenomenon coupled with the process of condensation. Within the framework of certain assumptions, the problem is recast to obtain a similarity variable solution for a practicable range of fin parameters. Subsequently it was shown by Wilkins [3] that an explicit solution is possible for the formulation of Patankar and Sparrow [2]. A significant conclusion of the article is that the studies of condensation on extended surfaces form a class by themselves and an estimation of the surface area requirements of the condenser by using classical Nusselt analysis for an isothermal case is not appropriate. Lienhard and Dhir [4] investigated the case of condensation on specific geometries for which similarity solutions can be obtained. However, surface thermal conditions were prescribed and the process of condensation was investigated for the assumed conditions of the surface. Poulikakos and Bejan [5], while investigating fin geometry for minimum entropy generation in forced convection, strongly advocated the optimization of extended surfaces based on energy-conservation philosophy.

In this note, the process of condensation on a vertical plate fin of variable thickness, is studied in order to establish the effect of fin geometry on the condensation heat transfer coefficient. The results would be of significance in subsequent studies on optimization of the fin geometry in connection with the development of flat plate heat pipes.

FORMULATION

The fin shown in Fig. 1 is exposed to quiescent saturated vapour and the base is maintained under isothermal conditions, at $T_w = T_{w,L}$. The fin is re-oriented differently from that studied by Patankar and Sparrow [2] to derive certain advantages in making the temperature one-dimensional and non-uniform along the spatial coordinate, x only, since $Lz \gg L$ and $t/L \ll 1$. The plate thickness, t, is assumed to vary according to the power law relationship

$$(t/t_0) = (x/L)^m$$

where m is a variable index. For thin condensate films, it is fairly established by Sparrow and Gregg [6] that the inertial forces of the condensate film do not contribute much and the sub-cooling effect is negligible. The contribution of the convective terms in the energy equation is fairly insignificant. In other words, using the classical assumptions of Nusselt, for the configuration shown in Fig. 1, the relevant equations can be written as shown below.

Conduction equation in the fin

$$\frac{d}{dx}\left[\left(\frac{x}{L}\right)^m \frac{dT_w}{dx}\right] + \frac{hP}{k_w A_s}(T_s - T_w) = 0. \quad (1)$$

Equation of conservation of momentum for the condensate film

$$\tau_w = \mu \left.\frac{du}{dy}\right|_{y=0} = \rho_L g_s \delta \quad (2)$$

where g_s is the component of the universal acceleration due to gravity, g, along the surface and is equal to $g \cos\phi$.

Energy balance associated with phase transformation is

$$\frac{d}{ds}\int_0^\delta u\,dy = \frac{k_L(T_s - T_w)}{\delta \rho_L h_{fg}} \quad (3)$$

where the variables y and s are measured normal and tangential to the extended surface, respectively (Fig. 1). For the geometry chosen, it can be shown that

$$\tan\phi = \frac{m}{2}\frac{t_0}{L}\eta^{m-1} \quad (4)$$

$$ds = dx(1+\tan^2\phi)^{1/2} \quad (5a)$$

$$\cos\phi = 1 - \frac{1}{2}\left[\frac{m}{2}\frac{t_0}{L}\eta^{m-1}\right]^2. \quad (5b)$$

The velocity and temperature fields in the condensate film are parabolic and linear, respectively, as per the assumptions. Thus

$$\frac{u}{u_1} = 2(y/\delta) - (y/\delta)^2 \quad (6)$$

$$q_w = k_L(T_s - T_w)/\delta. \quad (7)$$

Making use of equations (4)–(7), equations (1)–(3) can be written, respectively, in dimensionless form as follows:

$$\frac{d}{d\eta}\left[\eta^m \frac{dT^+}{d\eta}\right] - \frac{T^+}{\Delta} = 0 \quad (8)$$

$$U_1 = \frac{1}{2}\left[1 - \frac{1}{2}\tan^2\phi\right]\Delta^2 \quad (9)$$

$$\frac{d}{d\eta}[U_1\Delta] = \frac{3}{4}\frac{T}{M\Delta}\left[1 + \frac{1}{4}\tan^2\phi\right]. \quad (10)$$

NOMENCLATURE

A_s	cross-sectional area of the fin [m²]
C_p	specific heat at constant pressure [J kg⁻¹ K⁻¹]
E	efficiency of the fin (equation (16a))
g	acceleration due to gravity [m s⁻²]
g_s	component of g tangential to the fin surface
h	local heat transfer coefficient [W m⁻² K⁻¹]
h_{fg}	latent heat of condensation [J kg⁻¹]
h_m	average heat transfer coefficient [W m⁻² K⁻¹]
k	thermal conductivity [W m⁻¹ K⁻¹]
L	length of the fin [m]
m	index in equation, $t/t_0 = (x/L)^m$
M	fin parameter, $[Pr(h_{fg}/C_p\theta_{w,L})(gL^3/v^2)][2k_L L/k_w t_0]^4$
P	perimeter of the fin [m]
q	heat flux at the external surface [W m⁻²]
S	distance along the surface of the fin [m]
t	thickness of the fin at any x [m]
t_0	thickness at the base of the fin [m]
u_i	interfacial velocity of the condensate [m s⁻¹]
U_i	dimensionless velocity, $(\mu u_i/\rho^2 g L^2)[k_w t_0/2k_L L]^2$
x, y, z	spatial coordinates.

Greek symbols

δ	condensate film thickness [m]
Δ	dimensionless film thickness, $[\delta/L][k_w t_0/2k_L L]$
η	dimensionless space variable in the fin, x/L
μ	absolute viscosity of the condensate [kg m⁻¹ s⁻¹]
v	kinematic viscosity of the condensate [m² s⁻¹]
ρ_L	density of the condensate [kg m⁻³].

Dimensionless parameters

Nu	Nusselt number, hL/K
Nu_m	mean Nusselt number, $h_m L/K$
Pr	Prandtl number, $\mu C_p/k$
T^+	dimensionless temperature ratio, $(T_s - T_w)/(T_s - T_{w,L})$.

Subscripts

i	vapour–liquid interface
iso	isothermal conditions
L	liquid
s	saturation condition
w	wall
w,l	at the base of the fin.

With the aid of equation (9), U_i is eliminated from equations (8) and (10) giving the following constitutive differential equations:

$$\frac{d^2T^+}{d\eta^2} = \left[\frac{T^+}{P_1^{1/4}} - m\eta^{m-1}\frac{dT^+}{d\eta}\right]\eta^{-m} \tag{8'}$$

$$\frac{dP_1}{d\eta} = \frac{1}{F_1}\left[\frac{2T^+}{M}\left\{1 + \frac{A^2}{4}\eta^{2(m-1)}\right\} + \frac{4A^2P_1}{3}(m-1)\eta^{(2m-3)}\right] \tag{10'}$$

where

$$P_1 = \Delta^4;\quad F_1 = 1 - \frac{A^2}{2}\eta^{2(m-1)}\quad \text{and}\quad A = \frac{m}{2}\frac{t_0}{L}.$$

The boundary conditions are

$$U_i = P_1 = \frac{dT^+}{d\eta} = 0 \quad \text{at } \eta = 0 \tag{11}$$

$$T^+ = 1 \quad \text{at } \eta = 1. \tag{12}$$

Equations (8′) and (10′) are solved simultaneously subject to the boundary conditions given by equations (11) and (12). The local Nusselt number for condensation along the fin can be evaluated from the definition

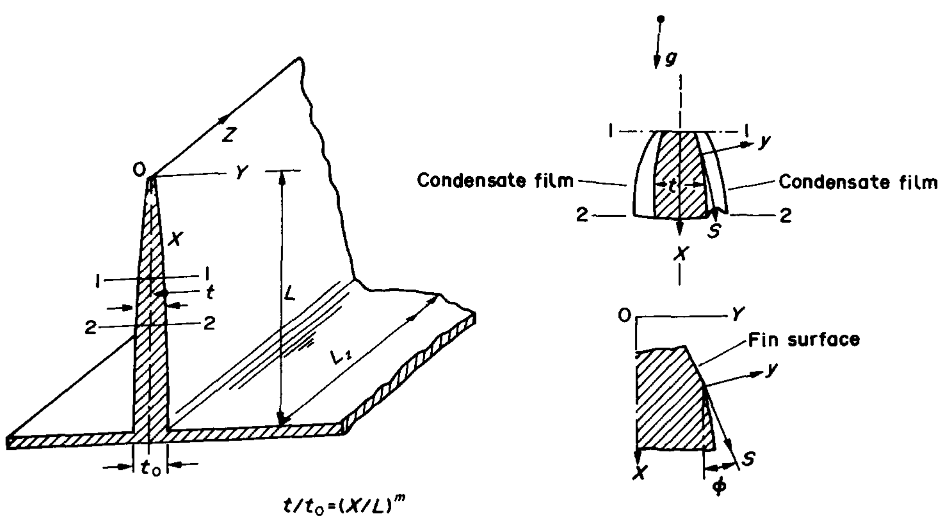
$$\frac{hL}{k_L} = \frac{L}{\delta} = \frac{1}{\Delta}\left[\frac{k_w t_0}{2k_L L}\right]. \tag{13}$$

Equation (13) is written in convenient form as

$$Nu = \frac{1}{\Delta M^{1/4}}\left[\frac{gL^3}{v^2}\frac{h_{fg}}{C_p\theta_{w,L}}Pr\right]^{1/4} = \frac{C}{\Delta M^{1/4}} \tag{14}$$

where

$$C = \left[\frac{gL^3}{v^2}\frac{h_{fg}}{C_p\theta_{w,L}}Pr\right]^{1/4}.$$



$t/t_0 = (X/L)^m$

$L_z \gg L$ Height of the fin ($t_0/L \ll 1$)

S = Tangent to the surface

y = Normal to the surface

FIG. 1. Configuration of the extended surface.

The average Nusselt number is given by

$$Nu_{\rm m} = \frac{1}{M^{1/4}}\left[\frac{gL^3}{\nu^2}\frac{h_{\rm fg}}{C_p\theta_{\rm w,L}}Pr\right]^{1/4}\int_0^1 \frac{1}{\Delta}\,{\rm d}\eta. \qquad (15)$$

The performance of the fin is further evaluated by introducing a definition for the efficiency of the fin as the ratio of the actual condensation taking place on the fin to the rate estimated from the classical Nusselt analysis for isothermal conditions of the surface maintained at its base temperature

$$E = -k_{\rm w}A_{\rm s}\left.\frac{{\rm d}T_{\rm w}}{{\rm d}x}\right|_{x=L}[h_{\rm iso}(T_{\rm s}-T_{\rm w,L})PL]^{-1} \qquad (16a)$$

or

$$E = \frac{1}{0.943}\frac{1}{M^{1/4}}\left.\frac{{\rm d}T^+}{{\rm d}\eta}\right|_{\eta=1}. \qquad (16b)$$

The classical Nusselt analysis for a vertical isothermal surface gives the relation for the average Nusselt number as

$$Nu_{\rm iso} = 0.943\left[\frac{gL^3}{\nu^2}\frac{h_{\rm fg}}{C_p\theta_{\rm w,L}}Pr\right]^{1/4}. \qquad (17)$$

RESULTS AND DISCUSSION

Equations (8′) and (10′) are solved by the fourth-order Runge–Kutta method using the HP-1000 computer system subject to boundary conditions, equations (11) and (12), to give the temperature field in the fin and the thickness of the condensate film. Typical graphs are shown plotted in Figs. 2–5 for different values of M, the fin parameter and for a given profile of the fin cross-section (i.e. m is constant). As the fin parameter M increases, the temperature of the tip tends to the saturation temperature T_s and the condensate film thickness decreases at a given location. An increase in the value of M physically signifies either an increase in length or a change in the material of the fin on which condensation occurs. This indicates that beyond a certain value of M ($M > 10^4$) not shown in Fig. 2, the temperature profiles are unique and identical. This condition refers to the case of an infinitely long fin. The fin parameter M is a product of two dimensionless terms; the first grouping is identical to that which appears in the classical Nusselt analysis and the second represents the ratio of the thermal conductivity of the fin material to the conductivity of the condensate. Further, the influence of m on the temperature distribution is shown plotted in Fig. 2 for $0.1 < M < 1000$. As is evident, the shape of the fin has an influence on the temperature profile and consequently on the condensate film thickness as shown in Fig. 3. An increase in M indicates a decrease in the volume of the fin material for a given system pressure and length of the fin. The results for the profile parameter, $m = 0$, represent a straight fin with parallel edges. Typical results for local condensation heat transfer coefficients are shown plotted in Fig. 4 for different values of the fin parameter, M, and shapes of the fin profiles. It is observed from the numerical results of equation (14) that for a given value of M, an increase in

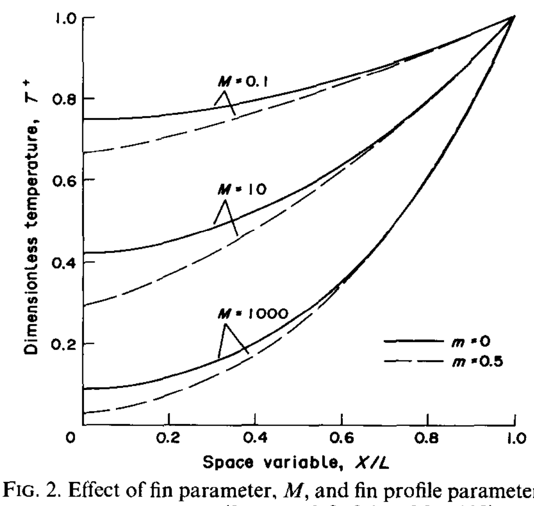


FIG. 2. Effect of fin parameter, M, and fin profile parameter, m, on temperature ($0 < m < 0.5$; $0.1 < M < 100$).

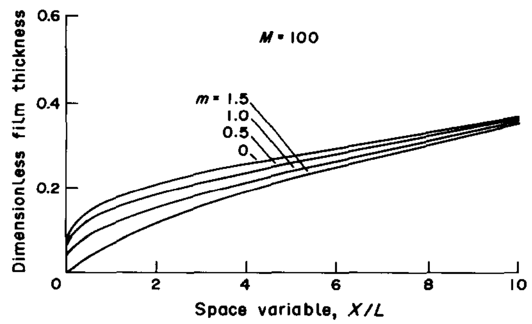


FIG. 3. Effect of fin profile geometry on the condensate film thickness ($0 < m < 1.5$ for $M = 100$).

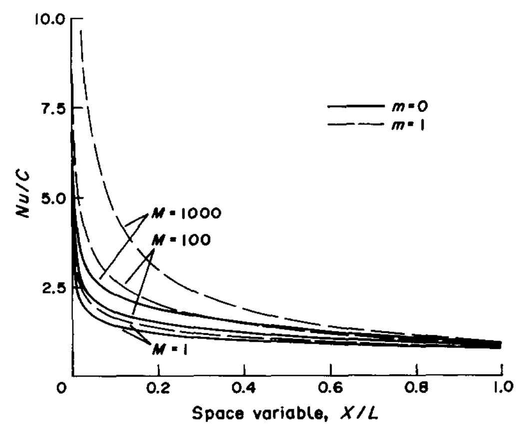


FIG. 4. Effect of fin parameter, M, and fin profile parameter, m, on local heat transfer coefficient.

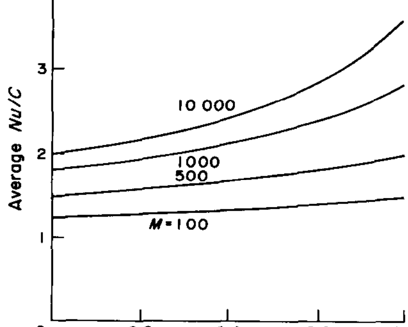


FIG. 5. Variation average heat transfer coefficients with fin profile geometry, m, for various fin parameters, M.

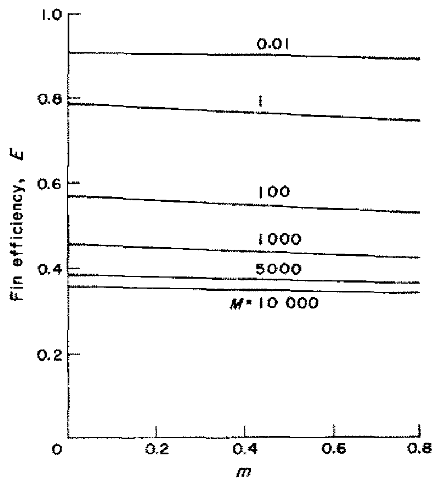


FIG. 6. Variation of fin efficiency with the fin profile geometry, m, for different fin parameters, M.

m gives rise to higher local heat transfer coefficients at the tip, whereas at the base, the heat transfer coefficients are asymptotically converging to a finite value. Similarly for a given shape of the profile of the fin chosen, an increase in the magnitude of the fin parameter, M, has resulted in a substantial increase in heat transfer coefficients. Equation (15) is shown plotted in Fig. 5 to depict the variation of the average Nusselt number for different configurations and fin parameters. It is obvious from the results that a reduction of the material of the fin would lead to thinner films resulting in higher values for the average heat transfer coefficients. Equation (16) reveals the efficiency of the fin. The results in Fig. 6 reveal that for chosen values of m, the efficiency seems to be strongly dependent on the fin parameter, M, and m has no perceptible influence for low values of M. However, in terms of the augmentation ratio defined as (h/h_{iso}) for any given location, the fin gives substantially higher heat transfer coefficients and the ratio is more than one. This aspect would be of utmost significance which affects the compactness of heat pipes employing fins either inside or outside of the heat pipe. In conclusion, this note gives salient results for a wide range of parameters related to the condensation phenomena on vertical extended surfaces of varying thickness, hitherto not solved.

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Effect of waves on Nusselt condensation

MAZHAR ÜNSAL†

Department of Mechanical Engineering, University of Hawaii at Manoa, Honolulu, Hawaii, U.S.A.

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INTRODUCTION

NUMEROUS analyses of steady laminar condensate films flowing under the action of gravity down a vertical isothermal plane surface and adjacent to a saturated quiescent vapour have appeared in the heat transfer literature. Analyses presented by Nusselt [1], Rohsenow [2], and Sparrow and Gregg [3] are examples of earlier original work on the subject.

Previous analyses of film condensation available in the literature have not focused attention on the effect of surface waves on the heat transfer through a condensate film and this subject is considered in the present study. The transient temperature field in a condensate film oscillating with an amplitude equal to the equilibrium wave amplitude predicted by the non-linear stability theory is utilized to compute a transient heat transfer coefficient. The transient heat transfer coefficient is then averaged with respect to time and space. The predicted average heat transfer coefficient is compared with the experimental data of Ritani and Shekriladze [4].

DISCUSSION

Previous results from the linear stability analysis of laminar film condensation [5] have shown that a laminar condensate film adjacent to a quiescent vapour is stable up to a critical distance from the leading edge of an isothermal vertical plate and unstable thereafter. The non-linear stability analysis in ref. [6] has shown that the linearly stable part of the film is also stable with respect to finite amplitude disturbances. The linearly unstable part of the condensate film, on the other hand, was found to reach finite equilibrium amplitudes provided that the Reynolds number is small and within the validity region of the long-wave perturbation analysis.

The analysis of the problem is outlined in the Appendix. Equations (4)–(6), (10) and (11) are simplified expressions valid for small F. It is noted that these formulas will apply to a wide range of situations since F is usually small in most practical applications. To the best knowledge of the author, an experimental study on the stability characteristics of condensate films is not available in the literature. Equations (4)–(6) are also valid for isothermal liquid films and comparison of the most unstable wave number and equilibrium amplitude with experiments are presented in Figs. 1–4 for the experimental data of Kapitza [7].

† On leave from the Department of Mechanical Engineering, University of Gaziantep, 27310 Gaziantep, Turkey.